*Massive Stars as Cosmic Engines*
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# Luminous Blue Variables & Mass Loss near the Eddington Limit

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**Abstract.** During the course of their evolution, massive stars lose a substantial fraction of their initial mass, both through steady winds and through relatively brief eruptions during their Luminous Blue Variable (LBV) phase. This talk reviews the dynamical driving of this mass loss, contrasting the line-driving of steady winds to the potential role of continuum driving for eruptions during LBV episodes when the star exceeds the Eddington limit. A key theme is to emphasize the inherent limits that self-shadowing places on line-driven mass loss rates, whereas continuum driving can in principle drive mass up to the "photon-tiring" limit, for which the energy to lift the wind becomes equal to the stellar luminosity. We review how the "porosity" of a highly clumped atmosphere can regulate continuum-driven mass loss, but also discuss recent time-dependent simulations of how base mass flux that exceeds the tiring limit can lead to flow stagnation and a complex, time-dependent combination of inflow and outflow regions. A general result is thus that porosity-mediated continuum driving in super-Eddington phases can explain the large, near tiring-limit mass loss inferred for LBV giant eruptions.

**Keywords.** stars: early-type, stars: winds, outflows, stars: mass loss, stars: activity

## 1. Introduction

Two key properties in making massive stars "cosmic engines" are their high luminosity, and their extensive mass loss. Indeed the momentum of this radiative luminosity is a key factor in driving massive-star mass loss, for example through the coupling with bound-bound opacity that is the basis of their more or less continuous line-driven stellar winds. Among the most luminous hot stars there appears a class of "Luminous Blue Variables" (LBVs) for which the winds are particularly strong, and exhibit irregular variability on time scales ranging from days to years. Contemporary observations generally suggest modest variations in net mass loss, occuring with nearly constant bolometric luminosity, and which might readily be explained by, e.g., opacity instabilities within the standard line-driving mechanism. But historical records, together with the extensive nebulae around many LBVs, suggest there are also much more dramatic eruptions, marked by substantial increases in the already extreme radiative luminosity, and lasting for several years, over which the net mass loss, $0.1 - 10\ M_{\odot}$, far exceeds what could be explained by line-driving. Rather, the closeness of such stars to the Eddington limit, for which the radiative force from just the electron scattering continuum would balance the force of gravity, suggests that such "giant eruptions" might instead arise from *continuum* driving, resulting in much higher mass loss, perhaps triggered by interior instabilities that increase the stellar luminosity above the Eddington limit.

The review here focusses on the underlying physical issues behind such historical LBV mass loss. One particular theme is whether such eruptions are best characterized as *explosions*, or as episodes of an enhanced quasi-steady *wind*. Key distinctions to be made include timescale (dynamic vs. diffusive), driving mechanism (gas vs. radiation pressure),

and degree of confinement (free expansion vs. gravitationally bound). As detailed below, it seems the characteristics of LBV giant eruptions require a combination of each, i.e. a quasi-steady wind driven by the enhanced lumosity associated with a relatively sudden (perhaps even explosive) release of energy in the interior. But even once an enhanced, super-Eddington luminosity is established, there remain fundamental issues of how the continuum driving can be regulated, e.g. by the spatial "porosity" of the medium, and thus lead to a mass loss that in some cases is inferred to have an energy comparable to the radiative luminosity, representing a "photon-tiring" limit.

## 2. The Key to Stellar Mass Loss: Overcoming Gravity

### 2.1. *Basic Momentum and Energy Requirements for Steady Wind*

Gravity is, of course, the essential force that keeps a star together as a bound entity, and so any discussion of stellar mass loss must necessarily focus on what mechanism(s) might be able to overcome this gravity. There are two aspects of this, namely to provide the momentum needed to reverse the inward pull of the gravitational force, but then also to have this outward driving sustained by tapping into a reservoir of energy that is sufficient to lift the material completely out of the star's gravitational potential.

For a steady radial wind flow, momenutum balance requires that any acceleration in speed $v$ with radius $r$ must result from a combination of the gradient of gas pressure with any other outward force to overcome the inward pull of gravity,

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dP}{dr} + g_{out}\,, \tag{2.1}$$

with standard notation for, e.g., mass density $\rho$ and stellar mass $M$. The discussion below focuses on radiative forces as a key to providing the required outward driving term $g_{out}$, but for now, let us just consider some general properties of such steady wind models.

First, at the base of any such wind outflow this momentum equation reduces to a hydrostatic equilibrium between the inward gravity and outward pressure,

$$-\frac{1}{P}\frac{dP}{dr} \equiv \frac{1}{H_P} = \frac{GM}{a^2 r^2}\,. \tag{2.2}$$

Here $a = \sqrt{kT/\mu}$ is the isothermal sound speed, with $k$ Boltzmann's constant and $\mu$ the mean molecular weight, and we have used the ideal gas law $P = \rho a^2$ to obtain an expression for the required local pressure scale height $H_P$.

The transition to a wind outflow occurs at some radius $R$ where the flow speed becomes supersonic, i.e. $v(R) = a$. In massive-star winds, for which the temperature is typically close to the stellar effective temperature, the sound speed $a \approx 20$ km/s, which is much less than the surface escape speed, $v_{esc} = \sqrt{2GM/R} \approx 600 - 1000$ km/s. This implies that from the sonic point outward, i.e. from $r > R$, gas pressure plays almost no role in maintaining the outward acceleration against gravity, reducing the momentum equation to

$$v\frac{dv}{dr} \approx -\frac{GM}{r^2} + g_{out} \quad ; \quad r \geqslant R\,. \tag{2.3}$$

Integration from this surface radius to infinity then immediately gives an expression for the required work per unit mass,

$$\int_R^\infty g_{out}\,dr \approx \frac{v(\infty)^2}{2} + \frac{GM}{R} = \frac{v_\infty^2}{2} + \frac{v_{esc}^2}{2}\,. \tag{2.4}$$

this ignores both the internal and kinetic energy at the sonic point, since these each are

only of order $a^2/v_{esc}^2 \approx 10^{-3}$ relative to the terms retained. For a wind with mass loss rate $\dot{M}$, the global rate of energy expended is then

$$L_{wind} = \dot{M} \left[ \frac{v_\infty^2}{2} + \frac{GM}{R} \right] . \tag{2.5}$$

The marginal case in which the wind escapes with vanishing terminal flow speed, $v_\infty = 0$, defines a minimum energy rate for lifting material to escape, $L_{min} = \dot{M}GM/R$. For a given available interior luminosity $L$, this thus implies a maximum possible, energy-limited mass loss rate

$$\dot{M}_{tir} = \frac{L}{GM/R} = 3.3 \times 10^{-8} \, \frac{M_\odot}{\mathrm{yr}} \left[ \frac{L}{M/R} \right] , \tag{2.6}$$

where the latter expression provides a convenient evaluation when the quantities in square brackets are written in solar units. The subscript here refers to reduction or "tiring" of the radiative luminosity as a result of the work done to sustain the outflow against gravity (Owocki & Gayley 1997). Even the most extreme massive-star steady winds, e.g. from WR stars, are typically no more than a few percent of this energy limit; but, as discussed further below, the mass loss during LBV giant eruptions can approach this order.

### 2.2. *Internal Energy and Virial Temperature*

Although gas pressure is not well-suited to driving a large steady mass loss from the stellar surface, it is generally the key to supporting the star against the inward pull of gravity. The associated pressure scale height is given locally by eqn. (2.2), which applied at the surface radius $r = R$ gives

$$\frac{H_p}{R} = \frac{2a_{eff}^2}{v_{esc}^2} \approx 10^{-3} , \tag{2.7}$$

where the latter scaling applies for a surface sound speed set by the stellar effective temperature, $a_{eff} \approx \sqrt{kT_{eff}/\mu}$.

However, for the stellar interior, the pressure drops from its central value to nearly zero at the surface, representing an average scale length $H_p \approx R$. This thus implies a characteristic interior sound speed $a_{int} \approx v_{esc}$, and a characteristic interior temperature

$$T_{int} \approx \frac{GM\mu}{kR} \approx 1.4 \times 10^7 \, K \, \frac{M}{R} , \tag{2.8}$$

where the ratio $M/R$ is in solar units, with $\mu \approx 10^{-24}$ g, roughly appropriate for fully ionized material of solar composition. This characteristic interior temperature can also be derived from the standard "virial theorum" result that the stellar internal thermal energy is half the gravitational binding energy, implying a negative net energy that keeps a star gravitationally bound.

But in the context of mass loss, it means that for gas pressure to have a sufficient internal energy to overcome gravity requires a temperature that is only a factor two larger than the typical interior value. In the solar corona, maintaining temperatures near this escape value does allow a pressure-driven solar wind, but this is only possible because the low density keeps the wind optically thin, with thus limited radiative cooling. For the much higher mass loss rates inferred for hot-star winds, the much higher density means that radiative cooling would prohibit ever reaching temperatures much above $T_{eff} \approx T_{int}/1000$. Thus, as noted above, gas pressure is simply not a viable mechanism for driving a dense, steady surface wind.

### 2.3. *Gas-Pressure-Driven Expansion in Dynamical Explosions*

On the other hand, gas pressure is indeed the primary driving mechanism for propelling the expansion from supernovae explosions. In this case, dynamical collapse of the stellar core of mass $M_{core} \approx M_{\odot}$ down to a radius characteristic of a neutron star or black hole, i.e. $R_{ns} \approx 10$ km/s, releases an energy

$$\Delta E \approx \frac{GM_c^2}{R_{ns}} \approx 10^{53}\mathrm{erg} \tag{2.9}$$

which is of order $10^4$ higher than the entire binding energy of the entire stellar envelope, $E_g \approx GM^2/R$. Transfer of just ca. 1% of this collapse energy can thus suddenly heat the surrounding stellar envelope to a temperature up to *hundred times* the equilibrium (virial) value, with an associated sound speed $a_{sn}$ up to ten times the gravitational escape speed. On a short dynamical time scale, $R/a_{sn}$, of order a few minutes, the associated large gas pressure then drives an accleration of the full envelope mass ($\sim 10M_{\odot}$) to a free expansion at speeds $v_{exp} \approx a_{sn}$, typically several thousand km/s.

The radiation generated by such SN explosions escapes on a somewhat longer time, with light curves typically peaking a few days after the initial explosion. But this is still significantly shorter than the characteristic time for LBV giant eruptions, which apparently can last for several years. Moreover, the expansion speeds inferred for LBV ejecta are typically a few hundred km/s, comparable to stellar escape speeds, and much less than the thousands of km/s typical for the initial expansion of supernovae. Thus, rather than a dynamical explosion wherein the gas overpressure simply overwhelms the binding from stellar gravity, it seems more likely that LBV eruptions may represent a quasi-controlled outburst, induced perhaps by an enhanced radiative brightening that leads to an outward radiative acceleration that exceeds gravity.

## 3. Radiatively Driven Mass Loss

### 3.1. *Radiative Acceleration and the Eddington Limit*

The force-per-unit mass imparted to material from interaction with radiation depends on an integration of the opacity and radiative flux over photon frequency $\nu$,

$$\mathbf{g}_{rad} = \int_0^{\infty} d\nu\, \kappa_\nu \mathbf{F}_\nu/c \equiv \kappa_F \mathbf{F}/c\,, \tag{3.1}$$

with $c$ the speed of light, and the latter equality defining the *flux-weighted* opacity $\kappa_F$ in terms of the bolometric radiative flux $\mathbf{F}$.

In general the opacity $\kappa_\nu$ includes both broad-band continuum processes – e.g. Thomson scattering of electrons, and bound-free or free-free absorption – and bound-bound transitions associated with line absorption and/or scattering. As discussed in §3.3, bound-bound opacity is most effective in near-surface layers where expansion from a not-too-dense wind can partially desaturate the strongest lines. But in a static envelope and atmosphere, the reduction in flux $F_\nu$ in such saturated lines keeps the associated line-force small, and so in most regions of a steller envelope the overall radiative acceleration is set by continuum processes like electron scattering and bound-free or free-free absorption.

In spherical symmetry, both the radial flux $F = L/4\pi r^2$ and gravity $g = GM/r^2$ have similar inverse-square dependence on radius $r$, which thus cancels in the ratio of radiative acceleration to gravity. In terms of the electron scattering opacity, $\kappa_e \approx 0.34\ cm^2/g$, this

ratio has the scaling

$$\Gamma \equiv \frac{g_{rad}}{g} = \frac{\kappa_F L}{4\pi G M c} = 2.6 \times 10^{-5} \, \frac{\kappa_F}{\kappa_e} \, \frac{L}{L_\odot} \, \frac{M_\odot}{M} \, . \tag{3.2}$$

When the opacity $\kappa$, radiative luminosity $L$, and mass $M$ are all fixed, then $\Gamma$ is constant. But, as discussed below, there are various circumstances in which this is not the case.

For pure electron scattering, with $\kappa_F = \kappa_e$, eqn. (3.2) just gives the classical Eddington parameter $\Gamma_e = \kappa_e L/4\pi G M c$. Because stellar luminosity generally scales with a high power of the stellar mass, i.e. $L \propto M^{3-4}$ (see §3.2), massive stars with $M > 10 M_\odot$ generally have electron Eddington parameters of order $\Gamma_e \approx 0.1 - 1$. Indeed, $\Gamma_e \equiv 1$ defines the *Eddington limit*, for which the entire star would formally become unbound, at least in this idealized model of 1-D, spherically symmetric, radiative envelope.

However, because the reversal of gravity formally extends to arbitrarily deep, dense layers of the stellar envelope, any outward mass flux that might be initiated would require a very large mechanical luminosity, and thus would be well above the energy, photon-tiring limit given in eqn. (2.6). As such, exceeding the Eddington limit does not represent an appropriate condition for the steady-state mass loss characteristic of a stellar wind, since that requires an outwardly increasing radiative force that goes from being less than gravity in a bound stellar envelope to exceeding gravity in the outflowing stellar wind. The discussion below summarizes how the necessary force regulation can still occur through line-desaturation for line driving (§3.3), and through porosity of spatial structure for continuum driving (§3.5).

But first let us briefly review the key scalings of stellar structure that lead massive stars to be so close to this fundamental Eddington limit.

### 3.2. *Stellar Structure Scaling for Luminosity vs. Mass*

The structure of a stellar envelope is set by the dual requirements for momentum balance and energy transport. The former is described through the equation for hydrostatic equilibrium (cf. eqn. 2.2), modified now to account for a factor $1 - \Gamma$ reduction in the effective gravity, due to the radiation force. Following the same approach as in §2.2, this thus now implies a characteristic interior temperature that scales as

$$T \sim \frac{M(1-\Gamma)}{R} \, . \tag{3.3}$$

Through most of the stellar envelope, the energy flux $F = L/4\pi r^2$ is tranported by diffusion of radiative energy density $U_{rad} \sim T^4$,

$$F = -\frac{1}{\kappa \rho c} \frac{dU_{rad}}{dr} \, , \tag{3.4}$$

which implies the dimensional scaling

$$L \sim \frac{R^4 T^4}{M} \, . \tag{3.5}$$

When combined with eqn. (3.3) for the interior temperature, we see that the *radius cancels* in the scaling of luminosity, yielding

$$L \sim M^3 \, (1-\Gamma)^4 \, . \tag{3.6}$$

Quite remarkably, this scaling does not depend explicitly on the nature of energy generation in the stellar core, but is strictly a property of the envelope structure†.

† Of course, this simple one-point scaling relation does have to be modified to accomodate

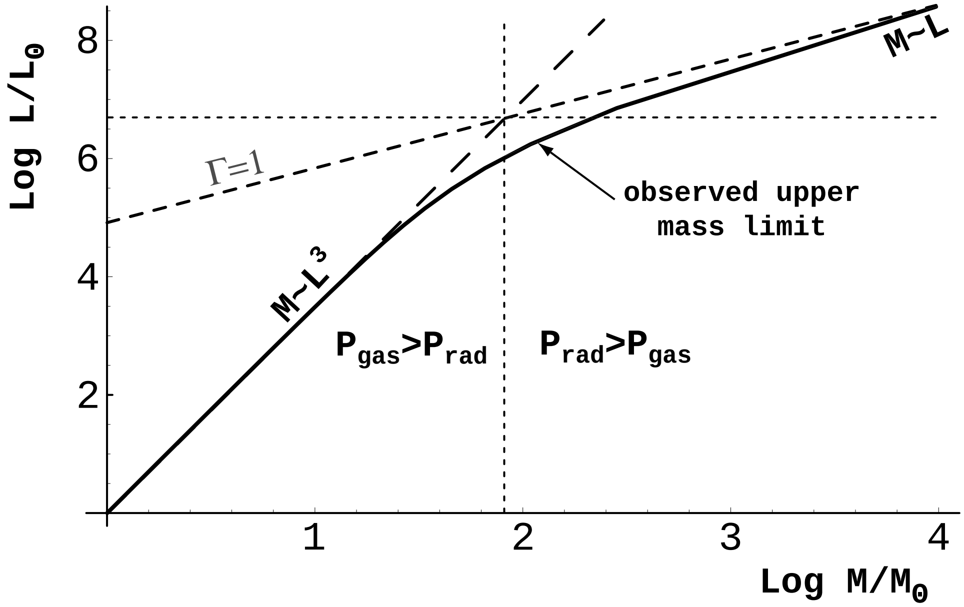


**Figure 1.** Log-log plot of the scaling of stellar luminosity $L$ vs. mass $M$ implied by the simple relation (3.6).

Figure 1 shows a log-log plot of the resulting variation of luminosity vs. mass. For low-mass stars, it implies a strong $L \sim M^3$ scaling, but as this forces stars to approach the Eddington limit, the $1-\Gamma$ term acts as a strong repeller away from that limit, causing a broad bend toward a linear asymptotic scaling, $L \sim M$.

Formally, this scaling suggests it is in principle possible to have stars with arbitrarily large mass, approaching arbitrarily close to the Eddington limit. But surveys of dense young clusters are providing increasingly strong evidence for a sharp cutoff in the stellar mass distribution at about $M \approx 150 - 200\ M_{\odot}$ (Oey and Clarke 2005; Kim et al. 2006).

Note that this inferred upper mass limit corresponds closely to the center of the bend region in fig. 1. This is just somewhat beyond the transition, at $\Gamma \approx 1/2$, to where radiation plays the dominant role in supporting the star against gravity, implying a radiation pressure that is greater than gas pressure, $P_{rad} > P_{gas}$. Somewhat analagous to having a heavier fluid support a lighter one, such a configuration may be subject to various kinds on intrinsic instabilities, leading to spatial clumping and/or the brightness variations that trigger LBV eruptions (Spiegel & Tau 1999; Shaviv 1998, 2000, 2001). The large associated LBV mass loss of such near Eddington stars thus could play a key role in setting the stellar upper mass limit.

### 3.3. *Line-Driven Stellar Winds*

The resonant nature of line (bound-bound) absorption leads to an opacity that is inherently much stronger than from free electrons. For example, in the somewhat idealized, optically thin limit that all the line opacity could be illuminated with a flat, unattenuated continuum spectrum with the full stellar luminosity, the total line-force would exceed the free-electron force by a huge factor, of order $Q \approx 2000$ (Gayley 1995). For massive stars

gradients in the molecular weight as a star evolves from the zero-age main sequence, and it breaks down altogether in the coolest stars (both giants and dwarfs), for which convection dominates the envelope energy transport.

with typical electron Eddington parameters within a factor two of unity, $\Gamma_e \approx 1/2$, this implies a net outward line acceleration that could be as high as $\Gamma_{lines} \approx Q\Gamma_e \approx 1000$ times the acceleration of gravity!

Of course, this does not generally occur in practice because of the self-absorption of the lines. For a single line with frequency-integrated opacity $\kappa_q = q\kappa_e$, the reduction in the optically thin line-acceleration $q\Gamma_e$ can be written as

$$\Gamma_{line} \approx q\Gamma_e \, \frac{1 - e^{-qt}}{qt} \, , \tag{3.7}$$

where $t \equiv \kappa_e \rho c/(dv/dr)$ is the Sobolev optical depth of a line with unit strength, $q = 1$ (Sobolev 1960; Castor, Abbott & Klein 1975, hereafter CAK). Within the standard CAK line-driven wind theory, the number distribution $N$ of spectral lines is approximated as a power law in line strength $q\, dN/dq = [1/\Gamma(\alpha)](q/Q)^{\alpha-1}$, where the CAK power index $\alpha \approx 0.5 - 0.7$ (and $\Gamma(\alpha)$ here represents the complete Gamma function). The associated line-ensemble-integrated radiation force is then reduced by a factor $1/(Qt)^\alpha$ from the optically thin value,

$$\Gamma_{lines} = \frac{Q\Gamma_e}{(1-\alpha)(Qt)^\alpha} \propto \left( \frac{1}{\rho} \frac{dv}{dr} \right)^\alpha \, . \tag{3.8}$$

The latter proportionality emphasizes the key scaling of the line-force with the velocity gradient $dv/dr$ and *inverse* of the density, $1/\rho$. This keeps the line acceleration less than gravity in the dense, nearly static atmosphere, but also allows its outward increase above gravity to drive the outflowing wind. The CAK mass loss rate is set by the associated critical density that allows the outward line acceleration to be just sufficient to overcome the (electron-scattering-reduced) gravity, i.e. with $\Gamma_{lines} \approx 1 - \Gamma_e$,

$$\dot{M}_{CAK} = \frac{\alpha}{1-\alpha} \, \frac{L}{c^2} \left[ \frac{Q\Gamma_e}{1-\Gamma_e} \right]^{-1+1/\alpha} \, , \tag{3.9}$$

where we have used the definition of the mass loss rate $\dot{M} \equiv 4\pi\rho v r^2$ and the fact that for such a CAK solution, $v dv/dr \approx g(1-\Gamma_e)$.

This last property further yields the characteristic CAK velocity law scaling $v(r) \approx v_\infty (1 - R/r)^{1/2}$, with the wind terminal speed being proportional to the effective surface escape speed,

$$v_\infty \propto v_{eff} \equiv \sqrt{GM(1-\Gamma_e)/R} \, . \tag{3.10}$$

As a star approaches the classical Eddington limit $\Gamma_e \to 1$, these standard CAK scalings formally predict the mass loss rate to diverge as $\dot{M} \propto 1/(1-\Gamma_e)^{(1-\alpha)/\alpha}$, but with a vanishing terminal flow speed $v_\infty \propto \sqrt{1-\Gamma_e}$. The former might appear to provide an explanation for the large mass losses inferred in LBV's, but the latter fails to explain the moderately high inferred ejection speeds, e.g. the 500-800 km/s kinematic expansion inferred for the Homunculus nebula of $\eta$ Carinae (Smith 2002, Smith et al. 2003).

But one essential point is that line-driving could never explain the extremely large mass loss rates needed to explain the Homunculus nebulae. To maintain the moderately high terminal speeds, the $\Gamma_e/(1-\Gamma_e)$ factor would have to be of order unity. Then for optimal realistic values $\alpha = 1/2$ and $Q \approx 2000$ for the line opacity parameters (Gayley 1995), the maximum mass loss from line driving is given by (Smith & Owocki 2006),

$$\dot{M} \approx 1.4 \times 10^{-4} L_6 \, M_\odot/yr \, , \tag{3.11}$$

where $L_6 \equiv L/10^6 L_\odot$. Even for peak luminosities of a few times $10^7 L_\odot$ during $\eta$ Carinae's eruption, this limit is still several orders of magnitude below the mass loss needed to form

the Homunculus. Thus, if mass loss during these eruptions occurs via a wind, it must be a super-Eddington wind driven by continuum radiation force (e.g., electron scattering opacity) and not lines (Owocki, Gayley & Shaviv 2004, hereafter OGS; Belyanin 1999; Quinn & Paczynski 1985).

### 3.4. *Convective Instability of a Super-Eddington Stellar Interior*

Before discussing such continuum-driven winds during periods of super-Eddington luminosity, it should first be emphasized that locally exceeding the Eddington limit need *not* necessarily lead to initiation of a mass outflow. As first shown by Joss, Salpeter, and Ostriker (1972), in the stellar envelope allowing the Eddington parameter $\Gamma \rightarrow 1$ generally implies through the Schwarzschild criterion that material becomes *convectively unstable*. Since convection in such deep layers is highly efficient, the radiative luminosity is reduced, thereby lowering the associated radiative Eddington factor away from unity.

This suggests that a radiatively driven outflow should only be initiated *outside* the region where convection is *efficient*. An upper bound to the convective energy flux is set by

$$F_{conv} \approx v_{conv}\, l\, dU/dr \lesssim a\, H\, dP/dr \approx a^3 \rho, \tag{3.12}$$

where $v_{conv}$, $l$, and $U$ are the convective velocity, mixing length, and internal energy density, and $a$, $H$, $P$, and $\rho$ are the sound speed, pressure scale height, pressure, and mass density. Setting this maximum convective flux equal to the total stellar energy flux $L/4\pi r^2$ yields an estimate for the maximum mass loss rate that can be initiated by radiative driving,

$$\dot{M} \leqslant \frac{L}{a^2} \equiv \dot{M}_{max,conv} = \frac{v_{esc}^2}{2a^2} \dot{M}_{tir}\,, \tag{3.13}$$

where the last equality emphasizes that, for the usual case of a sound speed much smaller than the local escape speed, $a \ll v_{esc}$, such a mass loss would generally be well in excess of the photon-tiring limit set by the energy available to lift the material out of the star's gravitational potential (see eqn. 2.6). In other words, if a wind were to originate from where convection becomes inefficient, the mass loss would be so large that it would use all the available luminosity to accelerate out of the gravitational potential, implying that any such outflow would necessarily stagnate at some finite radius. One can imagine that the subsequent infall of material would likely form a complex spatial pattern, consisting of a mixture of both downdrafts and upflows, perhaps even resembling the 3D cells of thermally driven convection.

Overall, it seems that a star that exceeds the Eddington limit is likely to develop a complex spatial structure, whether due to local instability to convection, to global instability of flow stagnation, or to intrinsic compressive instabilities arising from the dominance of radiation pressure.

### 3.5. *SuperEddington Outflow Moderated by Porous Opacity*

Shaviv (1998; 2000) has applied these notions of a spatially structured, radiatively supported atmosphere to suggest an innovative paradigm for how quasi-stationary wind outflows could be maintained from objects that formally exceed the Eddington limit. A key insight regards the fact that, in a laterally inhomogeneous atmosphere, the radiative transport should selectively avoid regions of enhanced density in favor of relatively low-density, "porous" channels between them. This stands in contrast to the usual picture of simple 1D, gray-atmosphere models, wherein the requirements of radiative equilibrium ensure that the radiative flux must be maintained independent of the medium's optical thickness. In 2D or 3D porous media, even a gray opacity will lead to a flux avoidance

of the most optically thick regions, much as in frequency-dependent radiative transfer in 1D atmosphere, wherein the flux avoids spectral lines or bound-free edges that represent a localized spectral regions of non-gray enhancement in opacity.

A simple description of the effect is to consider a medium in which material has coagulated into discrete blobs of individual optical thickness $\tau_b = \kappa \rho_b l$, where $l$ is the blob scale, and the blob density is enhanced compared to the mean density of the medium by a volume filling factor $\rho_b/\rho = (\text{Ł}/l)^3$, where Ł is the interblob spacing. The effective overall opacity of this medium can then be approximated as

$$\kappa_{eff} \approx \kappa \, \frac{1 - e^{-\tau_b}}{\tau_b} . \tag{3.14}$$

Note that in the limit of optically thin blobs ($\tau_b \ll 1$) this reproduces the usual microscopic opacity ($\kappa_{eff} \approx \kappa$); but in the optically thick limit ($\tau_b \gg 1$), the effective opacity is reduced by a factor of $1/\tau_b$, thus yielding a medium with opacity characterized instead by the blob cross section divided by the blob mass ($\kappa_{eff} = \kappa/\tau_b = l^2/m_b$). The critical mean density at which the blobs become optically thin is given by $\rho_o = 1/\kappa h$, where $h = \text{Ł}^3/l^2$ is characteristic "porosity length" parameter. A key upshot of this is that the radiative acceleration in such a gray, but spatially porous medium would likewise be reduced by a factor that depends on the mean density.

More realistically, it seems likely that structure should occur with a range of compression strengths and length scales. Noting the similarity of the single-scale and single-line correction factors (cf. eqns. 3.7 and 3.14), let us draw upon an analogy with the power-law distribution of line-opacity in the standard CAK model of line-driven winds, and thereby consider a *power-law-porosity* model in which the associated structure has a broad range of porosity length $h$. As detailed by OGS, this leads to an effective Eddington parameter that scales as

$$\Gamma_{eff} \approx \Gamma \left( \frac{\rho_o}{\rho} \right)^{\alpha_p} \quad ; \quad \rho > \rho_o \, , \tag{3.15}$$

where $\alpha_p$ is the porosity power index (analogous to the CAK line-distribution power index $\alpha$), and $\rho_o \equiv 1/\kappa h_o$, with $h_o$ now the porosity-length associated with the *strongest* (i.e. most optically thick) clump.

In rough analogy with the "mixing length" formalism of stellar convection, let us assume this porosity length $h_o$ scales with gravitational scale height $H \equiv a^2/g$. Then the requirement that $\Gamma_{eff} = 1$ at the wind sonic point yields a scaling for the mass loss rate scaling with luminosity. For the canonical case $\alpha_p = 1/2$, this takes the form (OGS),

$$\dot{M}_{por} \approx 4(\Gamma - 1) \, \frac{L}{ac} \, \frac{H}{h_o} \tag{3.16}$$

$$\approx 0.004(\Gamma - 1) \, \frac{M_\odot}{\text{yr}} \, \frac{L_6}{a_{20}} \, \frac{H}{h} \, . \tag{3.17}$$

The second equality gives numerical evaluation in terms of characteristic values for the sound speed $a_{20} \equiv a/20$ km/s and luminosity $L_6 \equiv L/10^6 L_\odot$. Comparision with the CAK scalings (3.9) for a line-driven wind shows that the mass loss can be substantially higher from a super-Eddington star with porosity-moderated, continuum driving. Applying the extreme luminosity $L \approx 20 \times 10^6 \, L_\odot$ estimated for the 1840-60 outburst of eta Carinae, which implies an Eddington parameter $\Gamma \approx 5$, the derived mass loss rate for a canonical porosity length of $h = H$ is $\dot{M}_{por} \approx 0.32 M_\odot/yr$, quite comparable to the inferred average $\sim 0.5 M_\odot/yr$ during this epoch.

Overall, it seems that, together with the ability to drive quite fast outflow speeds (of

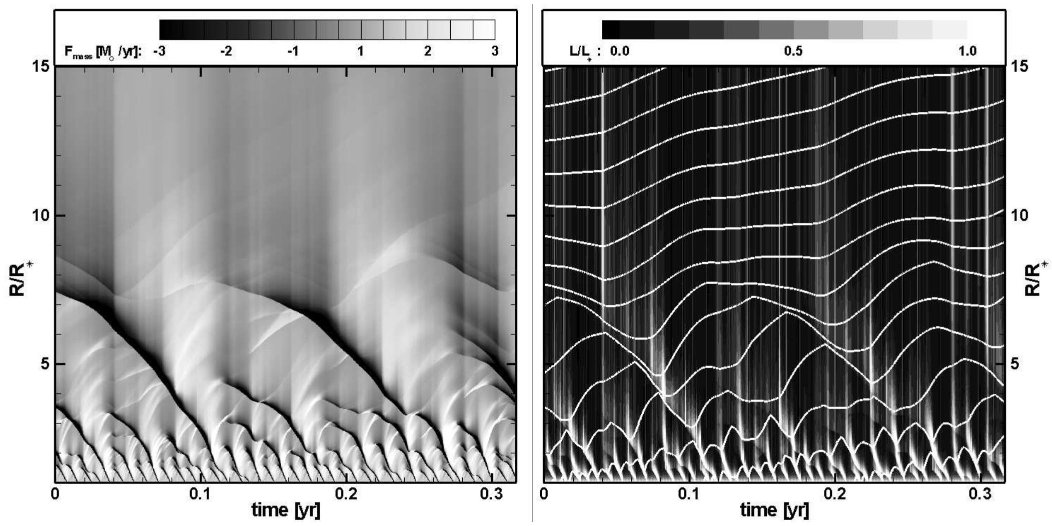


**Figure 2.** Grayscale plot of radius and time variation of mass flux (left) and luminosity (right) in time-dependent simulation of super-Eddington wind with porosity-mediated base mass flux above the photon tiring limit. The light contours on the right trace the height progression of fixed mass shells.

order the surface escape speed), the extended porosity formalism provides a promising basis for self-consistent dynamical modeling of even the most extreme mass loss outbursts of Luminous Blue Variables, namely those that, like the giant eruption of $\eta$ Carinae, approach the photon tiring limit.

### 3.6. *1D Simulation of Continuum-Driven Winds above the Photon-Tiring Limit*

For porosity models in which the base mass flux *exceeds* the photon tiring limit, recent numerical simulations (van Marle et al. 2007; see also poster I-49 in these proceedings) have explored the nature of the resulting complex pattern of infall and outflow. Despite the likely 3D nature of such flow patterns, to keep the computation tractable, this initial exploration assumes 1D spherical symmetry, though now allowing a fully time-dependent density and flow speed. The total rate of work done by the radiation on the outflow (or vice versa in regions of inflow) is accounted for by a radial change of the radiative luminosity with radius,

$$\frac{dL}{dr} = -\dot{m} g_{rad} = -\kappa_{eff}\, \rho v L/c \,, \tag{3.18}$$

where $\dot{m} \equiv 4\pi\rho v r^2$ is the local mass-flux at radius $r$, which is no longer a constant, or even monotonically positive, in such a time-dependent flow. The latter equality then follows from the definition (3.1) of the radiative acceleration $g_{rad}$ for a gray opacity $\kappa_{eff}$, set here by porosity-modified electron scattering. At each time step, eqn. (3.18) is integrated from an assumed lower boundary luminosity $L(R)$ to give the local radiative luminosity $L(r)$ at all radii $r > R$. Using this to compute the local radiative acceleration, the time-dependent equations for mass and momentum conservation are evolved forward to obtain the time and radial variation of density $\rho(r,t)$ and flow speed $v(r,t)$. (For simplicity, the temperature is fixed at the stellar effective temperature.) The base Eddington parameter is $\Gamma = 10$, and the analytic porosity mass flux is 2.3 times the tiring limit.

Figure 2 illustrates the flow structure as a function of radius (for $r = 1 - 15\, R$) and time (over an arbitrary interval long after the initial condition, set to analytic steady porosity

model ignoring photon tiring). The left panel grayscale shows the local mass flux, in $M_\odot$/ yr, with dark shades representing inflow, and light shades outflow. In the right panel, the shading represents the local luminosity in units of the base value, $L(r)/L(R)$, ranging from zero (black) to one (white); in addition, the superposed lines represent the radius and time variation of selected mass shells.

Both panels show the remarkably complex nature of the flow, with positive mass flux from the base overtaken by a hierarchy of infall from stagnated flow above. However, the re-energization of the radiative luminosity from this infall makes the region above have an outward impulse. The shell tracks thus show that, once material reaches a radius $r \approx 5R$, its infall intervals become ever shorter, allowing it eventually to drift outward. The overall result is a net, time-averaged mass loss through the outer that is very close to the photon-tiring limit, with however a terminal flow speed $v_\infty \approx 50$ km/s that is substantially below the surface escape speed $v_{esc} \approx 600$ km/s.

These initial 1D simulation thus provide an interesting glimpse into this competition below inflow and outflow. Of course, the structure in more realistic 2D and 3D models may be even more complex, and even lead itself to a highly porose medium. But overall, it seems that one robust property of such super-Eddington models may well be mass loss that is of the order of the photon tiring limit.

## 4. Conclusion

The basic conclusion of this review is that the extreme mass loss in giant eruptions of LBV stars seems best explained by quasi-steady, porosity-moderated, continuum-driven stellar wind during episodes of super-Eddington luminosity. The cause or trigger of this enhanced luminosity is unknown, but may be related to the dominance of radiation pressure over gas pressure in the envelopes of massive stars. The mass loss rate in such LBV eruptions is far greater than can be explained by the standard line-driving for hot-star winds in more quiescent phases. In the most massive stars, the cumulative mass loss in such eruptions may also dominate over the quiescent wind, and might even be a key factor in setting the stellar upper mass limit. Moreover, since driving by continuum scattering by free electrons does not directly depend on metalicity, mass loss by LBV eruptions may remain important in low-metalicity environments, including in the early universe. A key outstanding issue, however, is to determine the cause or trigger of the luminosity brightenings, including, for example, whether this might itself depend on metalicity.

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## Discussion

ZINNECKER: I completely agree with you that the term "radiation pressure" is ill-conceived, and we should better use a term like "radiative acceleration". I disagree with you, however, on another point: you were writing $L/M$ proportional to $M^2$ for very massive stars, when in reality it should be proportional to $M$ or even $M^{0.6}$ and approaching a constant near the Eddington limit; see my poster (# III-32) or Zinnecker & Yorke (2007), ARAA 45, 481. Or did I misunderstand you?

OWOCKI: Yes, I think there was some misunderstanding. The luminosity scalings you describe agree well with my simple envelope structure analysis. See eqn. (3.6) and fig. 1.